E.1 INTRODUCTION

Logistic regression (LR) was chosen as a tool to determine which of the factors identified in the mine-subsidence back analysis were the most significant with regard to predicting the probability of subsidence. Using data from selected subsided and unsubsided areas, the LR method gives estimates of model coefficients that can be used to quantify the probability of subsidence. The resulting regression model can then be used as a predictive tool in assessing subsidence hazard throughout the Tar Creek mining district.

Logistic regression is a statistical technique that has been extensively used in medical research to develop a relationship between a set of independent variables (regressors or explanatory variables) and a dependent variable (response variable) that is dichotomous (Ryan, 1997; Hosmer and Lemeshow, 2000). A dichotomous variable is one that has two possible states. In a medical study the independent dichotomous variable might be whether a person does or does not have a particular disease. The goal of a medical study might be to determine the contribution of age, weight, gender, and other factors to the probability of contracting a disease.

Logistic regression is also well suited to our study of mine subsidence as it allows us to estimate the probability that a particular mine will collapse based on a set of factors that we specify. In our analysis of mine failures we use a dichotomous variable indicating whether or not the mine has collapsed. The independent variables are drawn from a set of properties that we can determine from our understanding of the mechanics of mine failure and the availability of data. Once this set of variables has been specified, logistic regression provides a means of determining which factors are significant in predicting failure. This process involves analyzing pairs of stopes from the same mine, one which has failed, as evidenced by a subsidence feature, and one which has not failed. The resulting logistic regression coefficients can then be applied to other mines in the study area to estimate the probability that they will subsidence.

The choice of logistic regression as a tool to analyze our data set was driven by the nature of the event we are trying to predict. The event will either happen or not happen, and its occurrence depends upon several factors that we can measure.

Factors, or independent variables, that we chose to consider were based on an understanding of what contributes to a mine failure, and whether a mine had or had not subsided. The independent variables can be continuous, ordinal, or dichotomous. A continuous variable can take on any value in a range of values, such as the depth of a mine working. An ordinal variable can have one of a set of distinct categories, such as a geologic unit (alluvium, shale, or limestone) or degree of fracturing of a rock mass (high, medium, or low). Dichotomous variables were discussed previously. The method of selecting the independent variables for consideration is discussed in Section 6 of this report.

E.2 LOGISTIC REGRESSION

Following the development of Hosmer and Lemeshow (2000) we present the basic equations used in logistic regression. Let the dichotomous variable whose probability we want to predict be called Y, which depends on a set of independent variables X_i, where $i = 1, 2, ..., p$. A subsided mine is given a value of $Y = 1$, and an unsubsided mine is given a value of $Y = 0$. The probability that a mine will subsidence conditioned on a set of p independent variables $\mathbf{x}$ is given by

$$P(Y = 1 \mid \mathbf{x}) = \pi(\mathbf{x}), \qquad \text{(E-1)}$$

where $\mathbf{x} = (x_1, x_2, ..., x_p)^T$. We define the logit of the logistic regression model as

$$g(\mathbf{x}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p . \qquad \text{(E-2)}$$

The parameters β_i for $i = 1, 2, ..., p$ are called the model coefficients. The coefficient β_0 is referred to as the intercept, and the other coefficients are termed slopes. The probability that the mine will subsidence is related to the logit through the logistic transformation

$$\pi(\mathbf{x}) = \frac{1}{1 + e^{-g(\mathbf{x})}} . \quad \text{(E-3)}$$

The model coefficients β_i are determined by a maximum likelihood estimation procedure (Hosmer and Lemeshow, 2000). We used two commercially available statistical software packages (JMP and XLSTAT). All results reported here were computed using both packages and found to agree.

E.3 DESCRIPTION OF MODEL PARAMETERS

The parameters used to build the LR model come from the back analysis described in Section 6 of this report. All parameters were determined using data that came from two primary sources, mine maps and driller's logs, which are summarized in Tables E-1 and E-2, respectively. While other parameters might have been used to estimate the stability or instability of a rock mass, the set we chose was based on data availability and reliability.

For ease of recognition in the statistical analysis output, the model parameters are named by concatenating a sequential index and the parameter name. For example, the model parameter corresponding to the width of stope is labeled as 5Wst because it is the fifth parameter in our set of possible parameters.

The first two sets of independent variables came directly from mine maps and driller's logs. These are primarily dimensions and elevations of features in the mine workings. The third set of independent variables used in the LR model were derived from the ratios of variables from the first two sets. The rationale for using ratios is that some factors that contribute to subsidence might be scale independent and their importance might not be recognizable from their individual magnitudes alone. More importantly, the form of the logit (equation (E-2)) restricts models to linear combinations of parameters. This precludes possible nonlinear terms that may be essential to developing a satisfactory model. The ratio variables are described more fully in Table E-3.

The last set of variables corresponds to another class of derived parameters that we call stress surrogates. They are based on a model that uses a simple beam to approximate the stress in the mine roof. This model is described below.

E.4 ROOF-BEAM MODEL

A mine roof can be analyzed as a beam carrying a uniform load that is supported at its ends. The roof beam has a length L, a width W, and a vertical extent D (see Figure E-1). L and W refer to the dimensions of the portion of the mine roof between pillars that is under analysis. We assume that the material above the opening, which is loading the roof, is of uniform density ρ. We distinguish between material above the opening that contributes to the load and that which contributes to the strength of the roof. All geologic materials including surface chat piles, unconsolidated alluvium, fractured and weakened rock, and competent rock potentially contribute to the load. Their total thickness is called D_{load}. Competent geologic material, such as limestone in the Boone Formation and Chester (Quapaw and Heinzville Limestones), strengthen the roof. The total thickness of load bearing material is called $D_{strength}$. These two thicknesses are related through $D_{strength} \le D_{load}$, that is, all material may contribute to the load, while only some material contributes to the strength. This statement is not completely true in the case where a pressure arch forms above the opening, thereby shedding load onto surrounding material (Obert and Duval, 1967). The total load per unit length of the roof is given by

$$P = \rho W D_{load} . \quad \text{(E-4)}$$

The roof load produces a bending moment that arches the roof into a convex downward shape. Following traditional beam bending analysis (Case and Chilver, 1971), the maximum bending moment $M_{\max}$ occurs at the midpoint of a uniformly loaded beam and is given by

$$M_{\max} = \frac{PL_{span}^2}{8} . \quad \text{(E-5)}$$

The maximum compressive and extensional stresses occur at the top and bottom edges of the beam, respectively. They are related to the bending moment by

$$M_{\max} = \frac{2\sigma_{\max} I}{D_{strength}}, \tag{E-6}$$

where I is the moment of inertia of the beam about a horizontal axis through its center of gravity. For the rectangular cross-section beam forming the mine roof, the moment of inertia is

$$I = \frac{W\,D_{strength}^{3}}{12}. \tag{E-7}$$

Solving equation (E-6) for stress and substituting in equations (E-5) and (E-7) gives

$$\begin{aligned}\sigma_{\max} &= \frac{D_{strength}}{2}\frac{M_{\max}}{I} = \frac{D_{strength}}{2}\frac{PL_{span}^{2}}{8}\frac{12}{WD_{strength}^{3}} \\ &= \frac{3}{4}\frac{P}{W}\frac{L_{span}^{2}}{D_{strength}^{2}}.\end{aligned} \tag{E-8}$$

There are several problems with using beam analysis to estimate the stress in the mine roof. First, the density of the various units loading the mine roof are unknown and must be estimated. Second, we don't know the yield point of the geologic materials, so we do not know at what stress level the material fails. Third, the roof geometry is not as nice and neat as shown in Figure E-1, and, in fact, it may be more like a rectangular or irregularly shaped plate. Obert and Duval (1967) have analyzed a mine roof using a rectangular plate instead of a beam. For a plate, the actual stress levels will differ from those of a beam by a multiplicative factor, however, they will have the same dependence on the span and dimensions of the strength member. While we may not be able to determine the actual stress, we can determine a quantity that is proportional to stress, which we call a stress surrogate; it is given by

$$S = \frac{P}{W}\frac{L_{span}^{2}}{D_{strength}^{2}}. \tag{E-9}$$

The stress surrogate is proportional to the load per unit width (P/W) and the square of the span length, and inversely proportional to the square of the thickness of the strength member. We believe that the use of the stress surrogate based on a beam model is justified for our purposes of forming a physically meaningful variable to include in our regression analysis because failure of the mine roof must occur before any upward stoping of a cave-in can proceed.

We solve equation (E-4) for the load per unit width of the beam using the depth to the stope D_{st} for the thickness of the load,

$$\frac{P}{W} = \rho D_{st}, \tag{E-10}$$

where

$$\begin{aligned}D_{st} &= T_{cp} + T_{as} + T_{ch} + T_{bn} \\ &= T_{cp} + T_{as} + T_{rf}.\end{aligned} \tag{E-11}$$

T_{cp} is the thickness of any surface chat pile, T_{as} is the thickness of the combined alluvium and shale layers, T_{ch} is the thickness of the Chester, and T_{bn} is the thickness of the Boone formation. The combined Boone Formation and Chester thickness is called the roof thickness

$$T_{rf} = T_{ch} + T_{bn}. \tag{E-12}$$

The density of the various layers in equation (E-11) are not the same as shown in Table E-6. Accordingly, we use the sum of the products of the densities and thicknesses of the individual layers in equation (E-11) to obtain

$$\begin{aligned}\frac{P}{W} &= \rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{ch}T_{ch} + \rho_{bn}T_{bn} \\ &= \rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}.\end{aligned} \tag{E-13}$$

For the strength member we consider two cases: The first where only the Boone Formation and Chester are considered to be competent

$$D_{strength} = T_{bn} + T_{ch} = T_{rf}, \tag{E-14}$$

and the second where all geologic units contribute to the strength of the roof

$$D_{strength} = T_{as} + T_{rf}. \tag{E-15}$$

For the span of the roof, three cases are considered: the stope length L_{st}, the maximum unsupported stope span L_{un}, and the stope width W_{st}. The six stress surrogates are summarized in Table E-5.

One potential difficulty can be seen in the stress surrogates that have T_{rf} as the strength member. In some cases the roof thickness can be very small, in fact, taking on values of zero. When T_{rf} is zero, the stress surrogate becomes infinite. Even when, the roof thickness is non-zero, but very small, the stress surrogate may take on values that are more than the failure strength of the roof material. This situation will skew the data values used in the LR and lead to poor results. To prevent these situations from occurring, we modify slightly the stress surrogate as follows:

$$S = \min\left[\frac{P}{W}\frac{L_{span}^2}{\max\left(D_{strength}^2, 1\right)}, S_{\max}\right], \tag{E-16}$$

where $S_{\max}$ is some maximum allowed value to be determined. The maximum function in the denominator of equation (E-16) guarantees that there is no division by zero.

In Figure E-3 we plot histograms of the various stress surrogates. Extremely high values are not seen because the vertical axes were chosen to display the range of values that are considered to be reasonable. In most cases, 85 percent or more of the data points fall below the chosen values of $S_{\max}$.

E.5 MODEL BUILDING PARAMETER SELECTION

We have defined 28 parameters that will be considered for our LR model. This, however, is a situation where a philosophy of "the more, the merrier" is ill advisable as any attempt to use too many of these parameters in the model can be deleterious. One of the goals of model building is to find a set of explanatory variables that does a good job of predicting the data used to create the model. One way of increasing the predictive ability of a model is to increase the number of parameters in it. At first glance this seems like a good approach, as the resulting model does an excellent job of predicting the test data set, however, examination of the uncertainty in the model coefficients reveals that as the number of model parameters increases, so does the uncertainty in the parameter values. This phenomenon, referred to as over fitting, is to be avoided as it renders the regression model useless (Harrell et al., 1996).

Some of these candidate model parameters, such as the stress surrogates, were not tested in combination as they are trying to measure the same effect. In an attempt to sort through the parameters and get an initial estimate of their relative importance, a univariate (1-parameter) model was tried for each of the 28 parameters. A summary of these results is given in Table E-7.

Before discussing the results, we will give a brief description of the various statistical measures reported. Most of the statistics reported in Table E-7 are from a whole model test which compares the fit of the full model to a reduced model. The full model contains an intercept coefficient (β_0) and one slope coefficient (β_i) for each degree of freedom (DF) in the model. The reduced model contains only an intercept coefficient and no slope coefficients. The quantity negative log-likelihood of the difference ($-\log L_{diff}$) between the reduced and full model measures the significance of the regression model as a whole to fitting the data. Larger values of $-\log L_{diff}$ indicate a better fit; this quantity is reported in the third column.

The column labeled Chi-Sq is the likelihood-ratio chi-square test for the hypothesis that all regression parameters are zero. It is computed by taking twice the value of $-\log L_{diff}$. Again larger values are better. The p value in the next column is the probability of obtaining a greater chi-square value by chance alone if the specified model fits no better than the reduced model. R^2 is the ratio of the $-\log L$ for the difference to reduced models. It is sometimes called an uncertainty coefficient. It ranges from 0 to 1 corresponding to no improvement over the reduced model to a perfect fit, respectively.

The next column contains $-\log L_{full}$, which is the negative log-likelihood for the full model containing an intercept coefficient plus one coefficient for each degree of freedom. In this case the degrees of freedom (DF) is one. The last column contains the Akaike information criterion (Akaike, 1974), which is define as

$$AIC = -\log L_{full} + DF \,. \tag{E-17}$$

Smaller values of $-\log L_{full}$ indicate a better fit to the data, which can be obtained by adding additional degrees of freedom (more parameters) to a model. Unfortunately, this will eventually result in over fitting. AIC increases by one for each additional degree of freedom. This has the effect of penalizing AIC if too many parameters are added to the model. The smaller values of AIC correspond to better fits. AIC is one statistic that can be used to compare models with a different number of degrees of freedom.

ROC , reported in column 7, stands for receiver operating characteristics and is a diagnostic measure of model sensitivity and specificity. Sensitivity is the probability of a model correctly predicting that a subsided case exists (true positive). Specificity is the probability that a model correctly predicts that an unsubsided case exists (true negative). The quantity 1-specificity is the probability that the model incorrectly predicts that an unsubsided case is subsided. Sensitivity and specificity can be computed for different cutoff probability. If the computed LR probability exceeds the cutoff probability, the case is considered to be subsided. If the computed LR probability is less than the cutoff probability, the case is considered to be unsubsided. A plot of sensitivity against 1-specificity for different cutoff probabilities is called the ROC curve. If the predictive capability of a model is perfect, the area under the ROC curve will equal 1. For a completely random fit the area will equal 0.5. While larger values of ROC are better, they can be achieved by over fitting the data, resulting in a meaningless model, so additional criteria must be considered when values are close to 1.

Looking at Table E-7, which is sorted by decreasing value of AIC , we see that, in general, all of the various measures of model fit become better as AIC decreases. For example, the model predictability as measured by ROC increases as AIC decreases.

The best univariate regressor is T_{rf}/W_{st} . Intuitively, increasing roof thickness makes the mine roof stronger and less likely to subside, while roof deformation will increase as the stope becomes wider, thereby increasing the probability of subsidence. Using the ratio of these two parameters incorporates both of these characteristics into one parameter and makes the relative size of the parameters significant. The second best univariate regressor is T_{rf}/L_{un} , which is similar to the best regressor, except that the maximum unsupported span is used instead of the stope width. In both of these cases the ratio is a better estimator than a linear combination of the individual variables indicating that there is a scaling that must be taken into account. The third best regressor is the roof thickness. Of the 12 best regressors, six of them depend upon the thickness of the roof or the thickness of the Boone formation alone, while the other six are stress surrogates.

Individual measures of the mine dimensions such as height of stope (H_{st}), maximum unsupported span (L_{un}), extraction ratio, length of stope (L_{st}), and width of stope (W_{st}) are among the worst predictors. This indicates that the prediction of subsidence doesn't depend upon the magnitude of the values, but rather subsidence depends upon scaled variables.

In Figure E-4 we show the report from the JMP statistical software for parameter 17Tr/Wst. The Whole Model Test results have already been described above. The Lack of Fit (or goodness of fit) section of the report addresses the question of how effectively the model describes the dependent variable. The next section of the report gives the parameter estimates and their standard error. The chi-square values are the square of the ratio of the parameter estimate to its standard error. The larger the chi-square value the smaller the error estimate in the parameter. The p value associated with this chi-square value is given in the last column (labeled Prob>ChiSq). For this model, both p values are small indicating that the parameter estimates are quite reliable. The Effect Wald Tests result tests if the model with or without a particular parameter is better. These are the same values as in the last two columns of the parameter estimates section. The ROC information is given in the last section of the report. The best model discrimination corresponds to a curve that ascends vertically from the origin (0,0) to the point (0,1), and then goes horizontally to (1,1). In this ideal case the area under the ROC curve will equal 1. For the T_{rf}/W_{st} model the area is 0.8957, a very good value.

To develop two-parameter models, we started with the best one-parameter models and tried various combinations of parameters. Through trial and error it became obvious that parameter 17 (T_{rf}/W_{st}) was essential to produce a good model (see Table E-8). The addition of parameter 15 (H_{st}/D_{st}) resulted in a better model as shown by the various statistical measures in Figure E-5. The model gives a very good fit to the data, and all of the parameter estimates are very reliable. In addition the model discrimination as measured by ROC is excellent (0.9599).

The results of testing three-parameter models are shown in Figure E-9. Once again, the combination of parameters 17 and 15 along with others parameters resulted in three of the best models. The best three-parameter model included these two parameters plus the stress surrogate S_W.

E.6 OTHER PARAMETERS CONSIDERED AND REJECTED

In addition to the parameters described above, several other parameters were considered and rejected because there were insufficient data to use the parameter throughout the study area or the parameter was no better than a related parameter.

No attempt was made to analyze pillar stability because the exact shape and condition of pillars is uncertain. Pillar shape is shown on the mine maps, however, there is no indication as to whether or not the shape changes with distance between the floor and ceiling of the stope. Pillars are known to have been removed in some instances, and possibly there have been unreported removal of pillars. Inaccuracies in mine maps are another potential source of uncertainty in the state of pillars. Short of direct observation in the flooded mines by some borehole or autonomous sensing device, there is no easy way to obtain better information on pillar geometry and condition. Therefore, we have not tried to do any sort of analysis that requires such detailed information.

Likewise, we have rejected the use of rock mass quality or rock mass rating even though these methods can be of value. Though there are numerous drillers' logs throughout the mining district, the logs are based on cuttings and subject to the usual uncertainty of depth estimates for this drilling procedure. Furthermore, the log descriptions are not of a rigorous geologic nature. Interpretation of the terminology used by the drillers is required simply to estimate the depths of the geologic units in the area. Any attempt to assign a category to, much less quantify, fracture density using the drill logs would be misleading.

We did try to use hydraulic radius R_h, the area of the stope divided by its perimeter, as a model parameter

$$R_h = \frac{W_{st} L_{st}}{2\left(W_{st} + L_{st}\right)}, \tag{E-18}$$

because hydraulic radius has been used successfully in some analyses of underground opening stability. For the stopes examined in the study area there is a very good linear correlation between R_h and W_{st}. Accordingly, either parameter can be used in the LR with essentially the same result.

E.7 SUMMARY OF MODELS

The results of all of the models tested are summarized in Figure E-7. Plotted are the values of AIC as a function of the degrees of freedom (DF). Using the AIC the model with the smallest value and the small parameter standard error is considered the best. The best model for the various degrees of freedom are plotted as filled circles and connected by a solid line. The open circles are for models that either had larger values of AIC, or smaller values of AIC and large parameter standard errors. The best model is the three-parameter model using T_{rf}/W_{st}, H_{st}/D_{st}, and S_W, Physically these parameters make sense as good estimators of mine subsidence. As discussed earlier T_{rf}/W_{st} gives a measure of the relative strength to weakness of the roof rock versus the stope width. The second parameter, H_{st}/D_{st}, is related to whether or not a failure can stope to the surface. A roof fall followed by upward stoping that does not make it to the surface is not considered a subsidence in our analysis because we have seen no evidence of it. The last parameter S_W gives a measure of the stress in the roof rock based upon the loading from all units, strength from the roof rock alone, and the width of the stope (W_{st}). The width of the stope is an interesting parameter because it does not depend upon whether or not a pillar is missing or in poor condition. This is important as there is a great deal of uncertainty associated with the condition of pillars.

In Figure E-8 a scatter plot of the three best parameters is given for the subsided and unsubsided test cases. In general, the subsided and unsubsided data can be separated into two groups by a straight line as shown in the figure. The T_{rf}/W_{st} versus H_{st}/D_{st} plot shows the clearest separation of the data. This is to be expected because the model slopes (β_i) associated with these parameters have the smallest variances.

E.8 SELECTION AND RELIABILTY OF FINAL MODEL

Either the two- or three-parameter model described above should do a very good job predicting the probability of subsidence. The two-parameter model was chosen because of its simplicity. Use of either of the models, however, requires the determination of the stope width. As an automated method of determining W_{st} from the digitized mine maps has not been found, stope widths were determined manually and entered into the GIS database. This procedure is described in Section 6 and Appendix D of this report.

To demonstrate further the reliability of the two-parameter model to differentiate between subsided and unsubsided sites considered in the back analysis, we computed the probability of subsidence for all the back-analysis sites. Figure E-9 shows histograms of probability for the subsided and unsubsided sites. If a model worked perfectly, all of the subsided sites would have a low value of probability, while all of the subsided sites would have a high value of probability. A horizontal line set at a probability of 0.5 does a reasonable job of separating the data into subsided and unsubsided groups. Of the ten subsided cases, all but two are above the line, while for the unsubsided cases 14 out of 16 fall below the line. These histograms provide estimates of how well the LR model can discriminate between areas likely to subside and those unlikely to subside.

E.9 COMMENTS ON PROBABILITIES

For a collection of locations with the same values of the independent variable $\mathbf{x}$, the probability of subsidence multiplied by the total number of locations in the collection equals the number of locations that are expected to subside. It does not provide any information on which sites will and will not subside. Given two sites with the same probability values, they are equally likely to subside. If one site has a greater probability of subsidence than the other, the site with the higher probability is more likely to subside. However, this is not a guarantee that the site with the higher probability will subside and that the site with the lower probability will not subside.

Another aspect of the model that can cause some confusion is the fact that there is no time dependence in the model. While some general patterns of time dependence are suggested by the data, there is not sufficient evidence to establish a relationship. Accordingly, the probabilities apply to no specific time period. That is to say, we cannot make statements such as: The probability of subsidence in any given day, week, year, or decade is P.

In spite of these limitations, the probability estimates are very useful in screening areas at risk of subsidence from those that are less likely to subside. The probability maps that accompany this report show coherent patterns, that is, the high probability zones are well defined and not random in nature or based on a single anomalous point, lending confidence to the results.

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E.11 ACKNOWLEDGEMENT

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E-1 TABLES

Table E-1 Model parameters corresponding to independent variables determined from maps.

Index	Parameter	Symbol	Description/Comment
1	Levels		Number of mine levels
2	Shafts		Number of shafts entering stope
3	RockFall		If rock fall was mapped code as 1 otherwise code as 0. Not used in LR.
4	Tcp	T_{cp}	Height of surface chat pile, if present.
5	Wst	W_{st}	Width of stope
6	Lst	L_{st}	Length of stope
7	Lun	L_{un}	Maximum unsupported span of stope
8	Hst	H_{ST}	Floor to ceiling height of stope
9	Dst	D_{st}	Depth to roof of stope
10	Extract		Areal extraction ratio

Table E-2 Model parameters corresponding to independent variables determined from drillers' logs.

Index	Parameter	Symbol	Description/Comment
11	Tas	T_{as}	Thickness of alluvium and shale units above Chester
12	Tbn	T_{Bn}	Thickness of Boone Formation
13	Tch	T_{Ch}	Thickness of Chester
14	Trf	T_{rf}	Thickness of roof material; taken to be thickness of Boone Formation plus Chester

Table E-3 Model parameter corresponding to ratios of independent variables from the first two sets of independent variables.

Index	Parameter	Symbol	Description/Comment
15	Hst/Dst	H_{st}/D_{st}	height of stope:depth of stope
16	Lun/Dst	L_{un}/D_{st}	max unsupported stope span: depth of stope
17	Trf/Wst	T_{rf}/W_{st}	mine roof thickness:width of stope
18	Trf/Lun	T_{rf}/L_{un}	mine roof thickness:max unsupported span
19	Trf/Dst	T_{rf}/D_{st}	mine-roof thickness:depth of stope
20	Trf/Tas	T_{rf}/T_{as}	mine-roof thickness:thickness of alluvium+shale

Table E-4 Stress surrogate model parameters. See Table E-5 for complete description.

Index	Parameter	Symbol
21	S_L	S_L
22	S_U	S_U
23	S_W	S_W
24	S_LR	S_{LR}
25	S_UR	S_{UR}
26	S_WR	S_{WR}
27	S_LD	S_{LD}
28	S_UD	S_{UD}
29	S_WD	S_{WD}

Table E-5 Stress surrogates based on various measures of loading, strength, and span with densities included.

		L_{span}		
P/W	$D_{strength}$	L_{st} stope length	L_{un} unsupported length	W_{st} stope width
$\rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}$	T_{rf}	$S_L == \frac{\rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}}{T_{rf}^2} L_{st}^2$	$S_U == \frac{\rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}}{T_{rf}^2} L_{un}^2$	$S_W == \frac{\rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}}{T_{rf}^2} W_{st}^2$
$\rho_{rf}T_{rf}$	T_{rf}	$S_{LR} = \rho_{rf} \frac{L_{st}^2}{T_{rf}}$	$S_{UR} = \rho_{rf} \frac{L_{un}^2}{T_{rf}}$	$S_{WR} = \rho_{rf} \frac{W_{st}^2}{T_{rf}}$
$\rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}$	D_{st}	$S_{LD} = \frac{\rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}}{D_{st}^2} L_{st}^2$	$S_{UD} = \frac{\rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}}{D_{st}^2} L_{un}^2$	$S_{WD} = \frac{\rho_{cp}T_{cp} + \rho_{as}T_{as} + \rho_{rf}T_{rf}}{D_{st}^2} W_{st}^2$

Table E-6 Estimated unit weights and densities of the various geologic units overlying mine workings in the Tar Creek mining district.

Unit	Density [g/cm^3]	Unit weight [lb/ft^3]
chat	1.47	92
alluvium and shale	2.50	156
Chester	2.67	167
Boone Formation	2.67	167

Table E-7 Results of $DF = 1$ logistic regression for all model parameters sorted by decreasing AIC. Various columns are described in the text.

Index	Parameter	difference -log(L)	Chi-Sq	p	R^2	ROC	Full -log(L)	DF	AIC
5	Wst	0.00339	0.00678	0.9344	0.0002	0.5027	18.75686	1	19.75690
6	Lst	0.02289	0.04578	0.8306	0.0012	0.5481	18.73736	1	19.73740
10	Extract	0.03066	0.06132	0.8044	0.0016	0.5080	18.72959	1	19.72960
7	Lun	0.03973	0.07946	0.7780	0.0021	0.4759	18.72052	1	19.72050
4	Tcp	0.04517	0.09034	0.7637	0.0024	0.5241	18.71508	1	19.71510
27	S_LD	0.08427	0.16855	0.6814	0.0045	0.4840	18.67598	1	19.67600
8	Hst	0.14661	0.29322	0.5882	0.0078	0.5535	18.61364	1	19.61360
11	Tas	0.34107	0.68215	0.4088	0.0182	0.5989	18.41918	1	19.41920
29	S_WD	0.38658	0.77317	0.3792	0.0206	0.5241	18.37367	1	19.37370
28	S_UD	0.99176	1.98351	0.1590	0.0529	0.5989	17.76850	1	18.76850
13	Tch	1.18257	2.36514	0.1241	0.0630	0.6711	17.57768	1	18.57770
2	Shafts	1.34976	2.69952	0.1004	0.0719	0.6845	17.41049	1	18.41050
15	Hst/Dst	1.51720	3.03440	0.0815	0.0809	0.6658	17.24305	1	18.24310
16	Lun/Dst	1.63008	3.26016	0.0710	0.0869	0.6177	17.13017	1	18.13020
1	Levels	1.96837	3.93674	0.0472	0.1049	0.6925	16.79188	1	17.79190
9	Dst	2.09465	4.18929	0.0407	0.1117	0.7246	16.66560	1	17.66560
22	S_U	3.46192	6.92384	0.0085	0.1845	0.8556	15.29833	1	16.29830
23	S_W	3.48939	6.97878	0.0082	0.1860	0.8476	15.27086	1	16.27090
26	S_WR	4.06204	8.12409	0.0044	0.2165	0.8182	14.69821	1	15.69820
19	Trf/Dst	4.77562	9.55124	0.0020	0.2546	0.7995	13.98463	1	14.98460
12	Tbn	5.00668	10.01335	0.0016	0.2669	0.8209	13.75358	1	14.75360
21	S_L	5.12524	10.25048	0.0014	0.2732	0.8636	13.63501	1	14.63500
20	Trf/Tas	5.46901	10.93802	0.0009	0.2915	0.7995	13.29124	1	14.29120
24	S_LR	5.48657	10.97314	0.0009	0.2925	0.8155	13.27368	1	14.27370
25	S_UR	6.12431	12.24863	0.0005	0.3265	0.7968	12.63594	1	13.63590
14	Trf	7.07943	14.15887	0.0002	0.3774	0.8717	11.68082	1	12.68080
18	Trf/Lun	7.51897	15.03793	0.0001	0.4008	0.8636	11.24129	1	12.24130
17	Trf/Wst	8.06912	16.13824	0.0001	0.4301	0.8957	10.69113	1	11.69110

Table E-8 Results of $DF = 2$ logistic regression for all model parameters sorted by decreasing AIC.

Indices	Parameter	-log(L) diff	Chi-Sq	p	R^2	ROC	-log(L) full	DF	AIC
18 13	Trf/Lun Tch	7.56105	15.12210	0.0005	0.4030	0.8717	11.19920	2	13.19920
18 12	Trf/Lun Tbn	7.76516	15.53033	0.0004	0.4139	0.8663	10.99509	2	12.99510
18 25	Trf/Lun S_UR	7.80803	15.61607	0.0004	0.4162	0.8663	10.95222	2	12.95220
18 20	Trf/Lun Trf/Tas	7.96202	15.92404	0.0003	0.4244	0.8824	10.79823	2	12.79820
18 14	Trf/Lun Trf	8.05502	16.11003	0.0003	0.4294	0.8984	10.70524	2	12.70520
17 21	Trf/Wst S_L	8.07094	16.14188	0.0003	0.4302	0.9011	10.68931	2	12.68930
17 7	Trf/Wst Lun	8.07835	16.15669	0.0003	0.4306	0.8931	10.68191	2	12.68190
17 24	Trf/Wst S_LR	8.15468	16.30935	0.0003	0.4347	0.9011	10.60558	2	12.60560
17 19	Trf/Wst Trf/Dst	8.22972	16.45945	0.0003	0.4387	0.8957	10.53053	2	12.53050
17 1	Trf/Wst Levels	8.34952	16.69903	0.0002	0.4451	0.9118	10.41074	2	12.41070
18 21	Trf/Lun S_L	8.40897	16.81794	0.0002	0.4482	0.9144	10.35128	2	12.35130
17 12	Trf/Wst Tbn	8.43021	16.86041	0.0002	0.4494	0.8904	10.33005	2	12.33000
17 2	Trf/Wst Shafts	8.66454	17.32909	0.0002	0.4619	0.9064	10.09571	2	12.09570
17 20	Trf/Wst Trf/Tas	8.85726	17.71452	0.0001	0.4721	0.9118	9.90299	2	11.90300
17 10	Trf/Wst Extract	8.99730	17.99461	0.0001	0.4796	0.9305	9.76295	2	11.76290
17 18	Trf/Wst Trf/Lun	9.01735	18.03470	0.0001	0.4807	0.9144	9.74290	2	11.74290
17 25	Trf/Wst S_UR	9.02825	18.05651	0.0001	0.4812	0.9198	9.73200	2	11.73200
17 23	Trf/Wst S_W	9.04022	18.08043	0.0001	0.4819	0.9492	9.72004	2	11.72000
17 14	Trf/Wst Trf	9.15526	18.31053	0.0001	0.4880	0.9225	9.60499	2	11.60500
18 24	Trf/Lun S_LR	9.41130	18.82260	0.0001	0.5017	0.9251	9.34895	2	11.34900
17 9	Trf/Wst Dst	9.85406	19.70812	0.0001	0.5253	0.9332	8.90619	2	10.90620
17 15	Trf/Wst Hst/Dst	11.39037	22.78074	0.0001	0.6072	0.9599	7.36988	2	9.36988

Table E-9 Results of $DF = 3$ logistic regression for all model parameters sorted by decreasing AIC.

Indices	Parameters	-log(L) diff	Chi-Sq	p	R^2	ROC	-log(L) full	DF	AIC
10 14 20	Extract Trf Trf/Tas	7.50057	15.00114	0.0018	0.3998	0.8824	11.25968	3	14.25970
14 1 21	Trf Levels S_L	7.65333	15.30665	0.0016	0.4080	0.8957	11.10693	3	14.10690
14 10 21	Trf Extract S_L	8.57713	17.15426	0.0007	0.4572	0.8770	10.18312	3	13.18310
18 14 21	Trf/Lun Trf S_L	8.58558	17.17117	0.0007	0.4576	0.9198	10.17467	3	13.17470
24 25 21	S_LR S_UR S_L	8.77423	17.54846	0.0005	0.4677	0.9144	9.98602	3	12.98600
17 18 26	Trf/Wst Trf/Lun S_WR	9.01738	18.03476	0.0004	0.4807	0.9144	9.74287	3	12.74290
17 18 21	Trf/Wst Trf/Lun S_L	9.02815	18.05630	0.0004	0.4812	0.9198	9.73210	3	12.73210
17 18 29	Trf/Wst Trf/Lun S_WD	9.18010	18.36019	0.0004	0.4893	0.9198	9.58016	3	12.58020
17 18 28	Trf/Wst Trf/Lun S_UD	9.21672	18.43343	0.0004	0.4913	0.9251	9.54354	3	12.54350
17 18 25	Trf/Wst Trf/Lun S_UR	9.23136	18.46272	0.0004	0.4921	0.9198	9.52889	3	12.52890
17 14 21	Trf/Wst Trf S_L	9.25405	18.50810	0.0003	0.4933	0.9225	9.50620	3	12.50620
17 18 14	Trf/Wst Trf/Lun Trf	9.31763	18.63527	0.0003	0.4967	0.9251	9.44262	3	12.44260
17 18 27	Trf/Wst Trf/Lun S_LD	9.47490	18.94980	0.0003	0.5051	0.9198	9.28535	3	12.28540
17 18 24	Trf/Wst Trf/Lun S_LR	9.54857	19.09715	0.0003	0.5090	0.9144	9.21168	3	12.21170
14 2 21	Trf Shafts S_L	9.63677	19.27354	0.0002	0.5137	0.9332	9.12348	3	12.12350
14 2 25	Trf Shafts S_UR	9.77788	19.55576	0.0002	0.5212	0.9091	8.98237	3	11.98240
14 1 24	Trf Levels S_LR	9.78645	19.57289	0.0002	0.5217	0.9225	8.97381	3	11.97380
17 14 24	Trf/Wst Trf S_LR	9.84708	19.69415	0.0002	0.5249	0.9332	8.91318	3	11.91320
9 12 17	Dst Tbn Trf/Wst	9.85836	19.71671	0.0002	0.5255	0.9332	8.90189	3	11.90190
17 9 7	Trf/Wst Dst Lun	9.93878	19.87756	0.0002	0.5298	0.9305	8.82147	3	11.82150
17 9 11	Trf/Wst Dst Tas	9.94098	19.88197	0.0002	0.5299	0.9492	8.81927	3	11.81930
17 18 9	Trf/Wst Trf/Lun Dst	10.00805	20.01609	0.0002	0.5335	0.9358	8.75221	3	11.75220
17 8 14	Trf/Wst Hst Trf	10.01989	20.03978	0.0002	0.5341	0.9171	8.74036	3	11.74040
9 14 17	Dst Trf Trf/Wst	10.08683	20.17366	0.0002	0.5377	0.9332	8.67342	3	11.67340
17 9 21	Trf/Wst Dst S_L	10.09627	20.19253	0.0002	0.5382	0.9385	8.66399	3	11.66400
17 9 13	Trf/Wst Dst Tch	10.17571	20.35142	0.0001	0.5424	0.9385	8.58454	3	11.58450
9 13 17	Dst Tch Trf/Wst	10.17571	20.35142	0.0001	0.5424	0.9385	8.58454	3	11.58450
18 14 24	Trf/Lun Trf S_LR	10.19515	20.39029	0.0001	0.5434	0.9198	8.56511	3	11.56510
17 18 10	Trf/Wst Trf/Lun Extract	10.25800	20.51599	0.0001	0.5468	0.9251	8.50226	3	11.50230

Table E-9 continued Results of $DF = 3$ logistic regression.

Indices	Parameters	-log(L) diff	Chi-Sq	p	R^2	ROC	-log(L) full	DF	AIC
17 9 24	Trf/Wst Dst S_LR	10.29135	20.58269	0.0001	0.5486	0.9278	8.46891	3	11.46890
17 9 25	Trf/Wst Dst S_UR	10.30884	20.61768	0.0001	0.5495	0.9465	8.45141	3	11.45140
17 18 22	Trf/Wst Trf/Lun S_U	10.62229	21.24459	0.0001	0.5662	0.9198	8.13796	3	11.13800
17 18 23	Trf/Wst Trf/Lun S_W	10.63669	21.27338	0.0001	0.5670	0.9358	8.12356	3	11.12360
14 24 12	Trf S_LR Tbn	10.64368	21.28737	0.0001	0.5674	0.9439	8.11657	3	11.11660
14 2 24	Trf Shafts S_LR	11.16204	22.32408	0.0001	0.5950	0.9492	7.59821	3	10.59820
10 14 17	Extract Trf Trf/Wst	11.24796	22.49592	0.0001	0.5996	0.9412	7.51229	3	10.51230
17 8 9	Trf/Wst Hst Dst	11.42395	22.84790	0.0001	0.6089	0.9492	7.33630	3	10.33630
17 15 8	Trf/Wst Hst/Dst Hst	12.46135	24.92270	0.0001	0.6642	0.9652	6.29890	3	9.29890
17 15 10	Trf/Wst Hst/Dst Extract	12.60257	25.20515	0.0001	0.6718	0.9733	6.15768	3	9.15768
17 15 23	Trf/Wst Hst/Dst S_W	13.54840	27.09680	0.0001	0.7222	0.9759	5.21185	3	8.21185

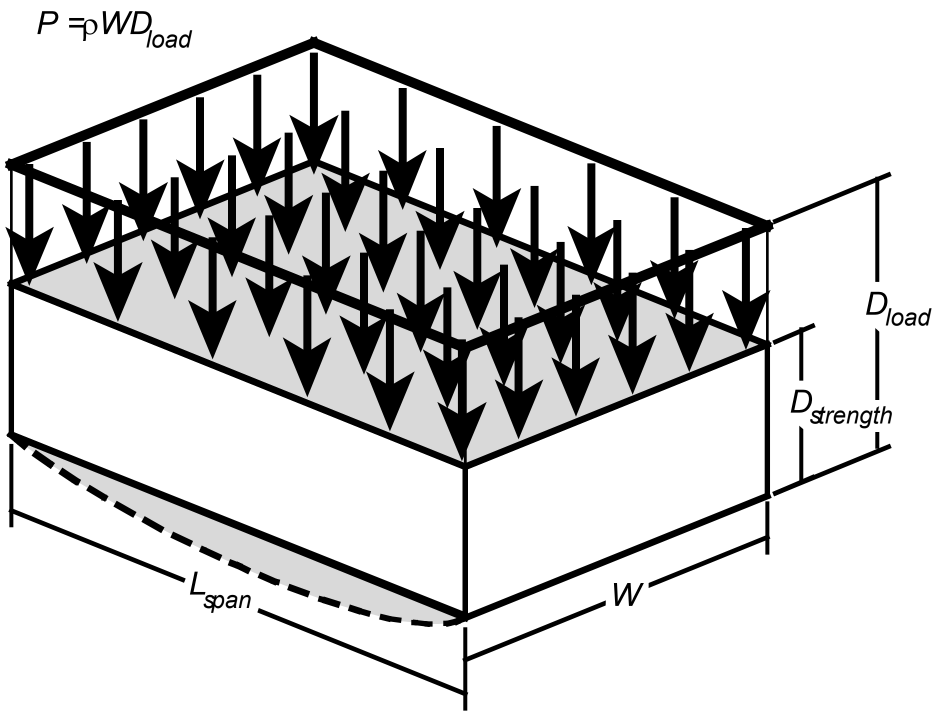


Figure E-1 Geometry of beam forming mine roof and load from material above opening. Deformation is assumed to occur along the span direction.

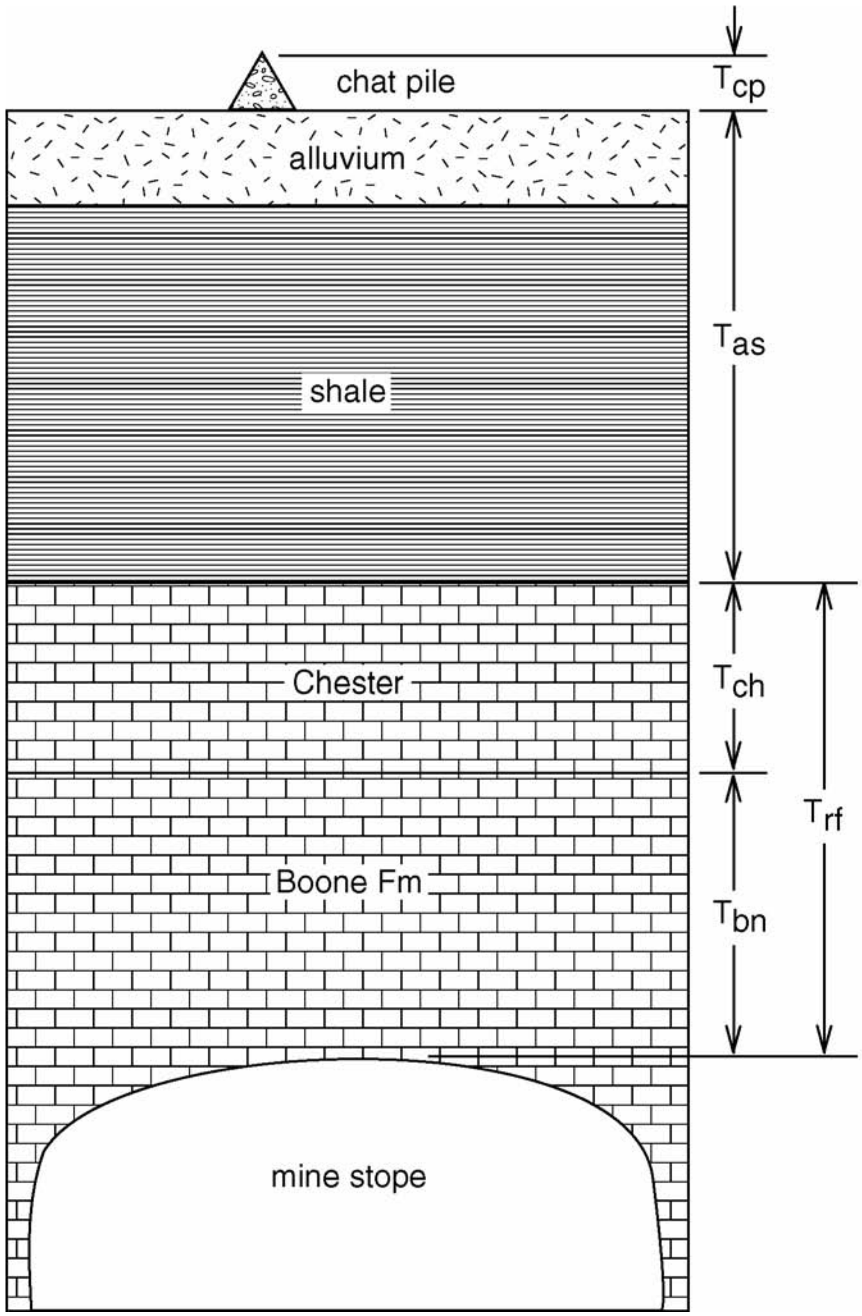


Figure E-2 Geologic units from Tar Creek mining district used in stress surrogate calculation. Vertical dimensions are roughly in proportion to average unit thicknesses. Not drawn to scale.

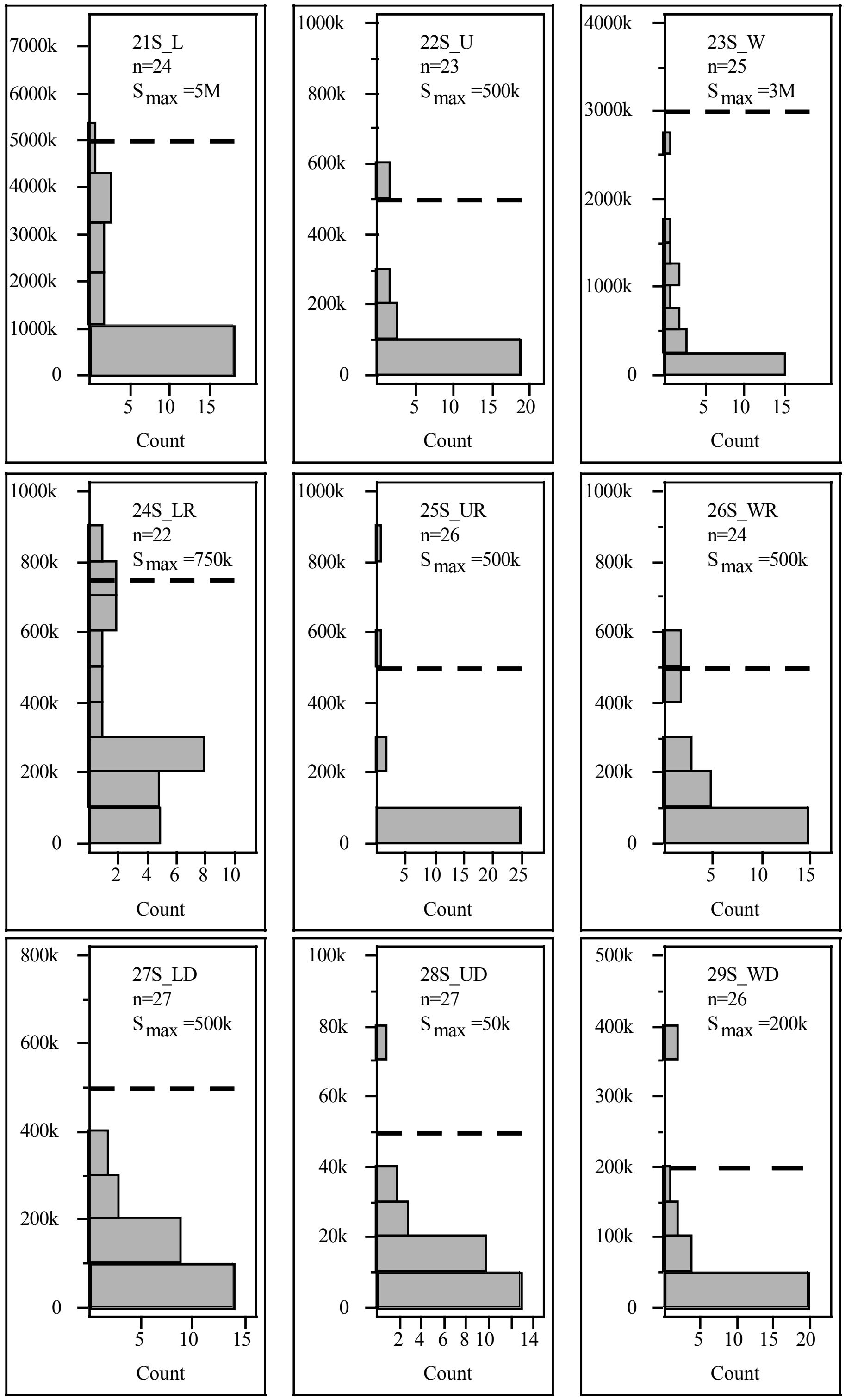


Figure E-3 Histograms of stress surrogates. The units of the stress surrogate are [lbs/ft^2]. The cutoff level, $S_{\max}$, is shown by a dashed line. The number of data points less than the cutoff level are shown. Total number of data points is 28.

Nominal Logistic Fit – Model 17Trf/Wst

Whole Model Test

Model	–LogLikelihood	DF	ChiSquare	Prob>ChiSq
Difference	8.069118	1	16.13824	<.0001
Full	10.691134			
Reduced	18.760251			

RSquare (U)	0.4301
Observations (or Sum Wgts)	28

Converged by Gradient

Lack Of Fit

Source	DF	–LogLikelihood	ChiSquare
Lack Of Fit	25	9.304839	18.60968
Saturated	26	1.386294	**Prob>ChiSq**
Fitted	1	10.691134	0.8154

Parameter Estimates

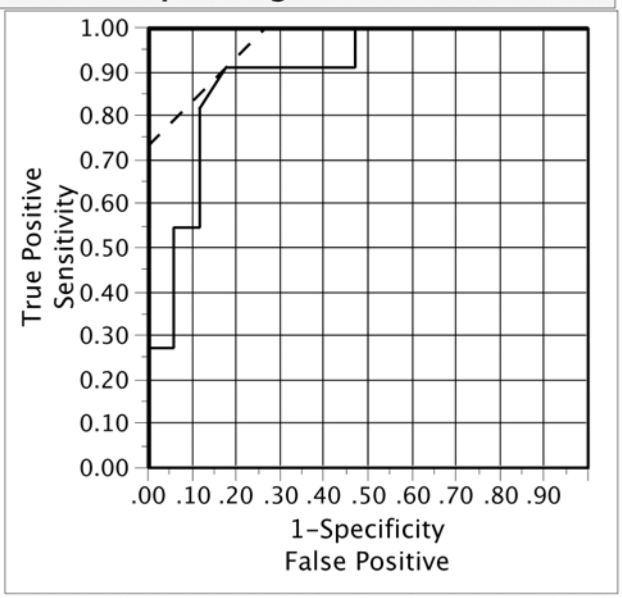
Term	Estimate	Std Error	ChiSquare	Prob>ChiSq
Intercept	2.65155889	1.130713	5.50	0.0190
17Trf/Wst	–10.766766	4.1385663	6.77	0.0093

For log odds of 0/1

Effect Wald Tests

Source	Nparm	DF	Wald ChiSquare	Prob>ChiSq
17Trf/Wst	1	1	6.7681616	0.0093

Receiver Operating Characteristic



Using Collapse no SH='1' to be the positive level
Area Under Curve = 0.89572

Figure E-4 Logistic regression output from JMP software for best $DF = 1$ model.

Nominal Logistic Fit – Model 17Trf/Wst 15Hst/Dst

Whole Model Test

Model	–LogLikelihood	DF	ChiSquare	Prob>ChiSq
Difference	11.390372	2	22.78074	<.0001
Full	7.369880			
Reduced	18.760251			

RSquare (U)	0.6072
Observations (or Sum Wgts)	28

Converged by Gradient

Lack Of Fit

Source	DF	–LogLikelihood	ChiSquare
Lack Of Fit	24	5.9835851	11.96717
Saturated	26	1.3862944	**Prob>ChiSq**
Fitted	2	7.3698795	0.9803

Parameter Estimates

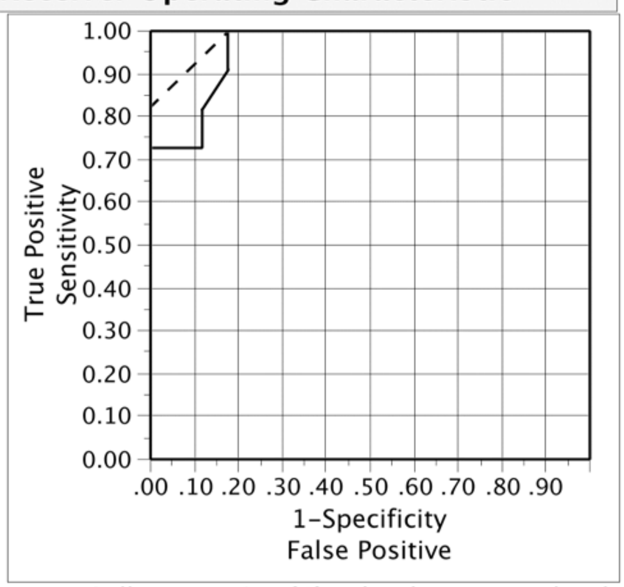
Term	Estimate	Std Error	ChiSquare	Prob>ChiSq
Intercept	1.71044982	1.5015088	1.30	0.2546
17Trf/Wst	–20.936947	9.1188146	5.27	0.0217
15Hst/Dst	9.1589063	4.7608986	3.70	0.0544

For log odds of 0/1

Effect Wald Tests

Source	Nparm	DF	Wald ChiSquare	Prob>ChiSq
17Trf/Wst	1	1	5.27169066	0.0217
15Hst/Dst	1	1	3.70091721	0.0544

Receiver Operating Characteristic



Using Collapse no SH='1' to be the positive level
Area Under Curve = 0.95989

Figure E-5 Logistic regression output from JMP software for best $DF = 2$ model.

Nominal Logistic Fit – Model 17Trf/Wst 15Hst/Dst 23S_W

Whole Model Test

Model	–LogLikelihood	DF	ChiSquare	Prob>ChiSq
Difference	13.548398	3	27.0968	<.0001
Full	5.211854			
Reduced	18.760251			

RSquare (U)	0.7222
Observations (or Sum Wgts)	28

Converged by Gradient

Lack Of Fit

Source	DF	–LogLikelihood	ChiSquare
Lack Of Fit	23	3.8255591	7.651118
Saturated	26	1.3862944	**Prob>ChiSq**
Fitted	3	5.2118535	0.9989

Parameter Estimates

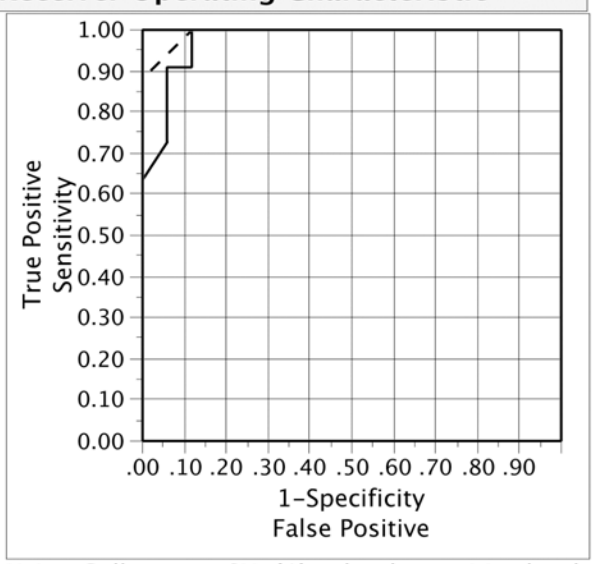
Term	Estimate	Std Error	ChiSquare	Prob>ChiSq
Intercept	9.53876108	5.3862201	3.14	0.0766
17Trf/Wst	–47.609393	21.466952	4.92	0.0266
15Hst/Dst	13.8271164	6.2460059	4.90	0.0268
23S_W	–0.0000033	0.0000021	2.62	0.1057

For log odds of 0/1

Effect Wald Tests

Source	Nparm	DF	Wald ChiSquare	Prob>ChiSq
17Trf/Wst	1	1	4.91863399	0.0266
15Hst/Dst	1	1	4.90070386	0.0268
23S_W	1	1	2.61750593	0.1057

Receiver Operating Characteristic



Using Collapse no SH='1' to be the positive level
Area Under Curve = 0.97594

Figure E-6 Logistic regression output from JMP software for best $DF = 3$ model.

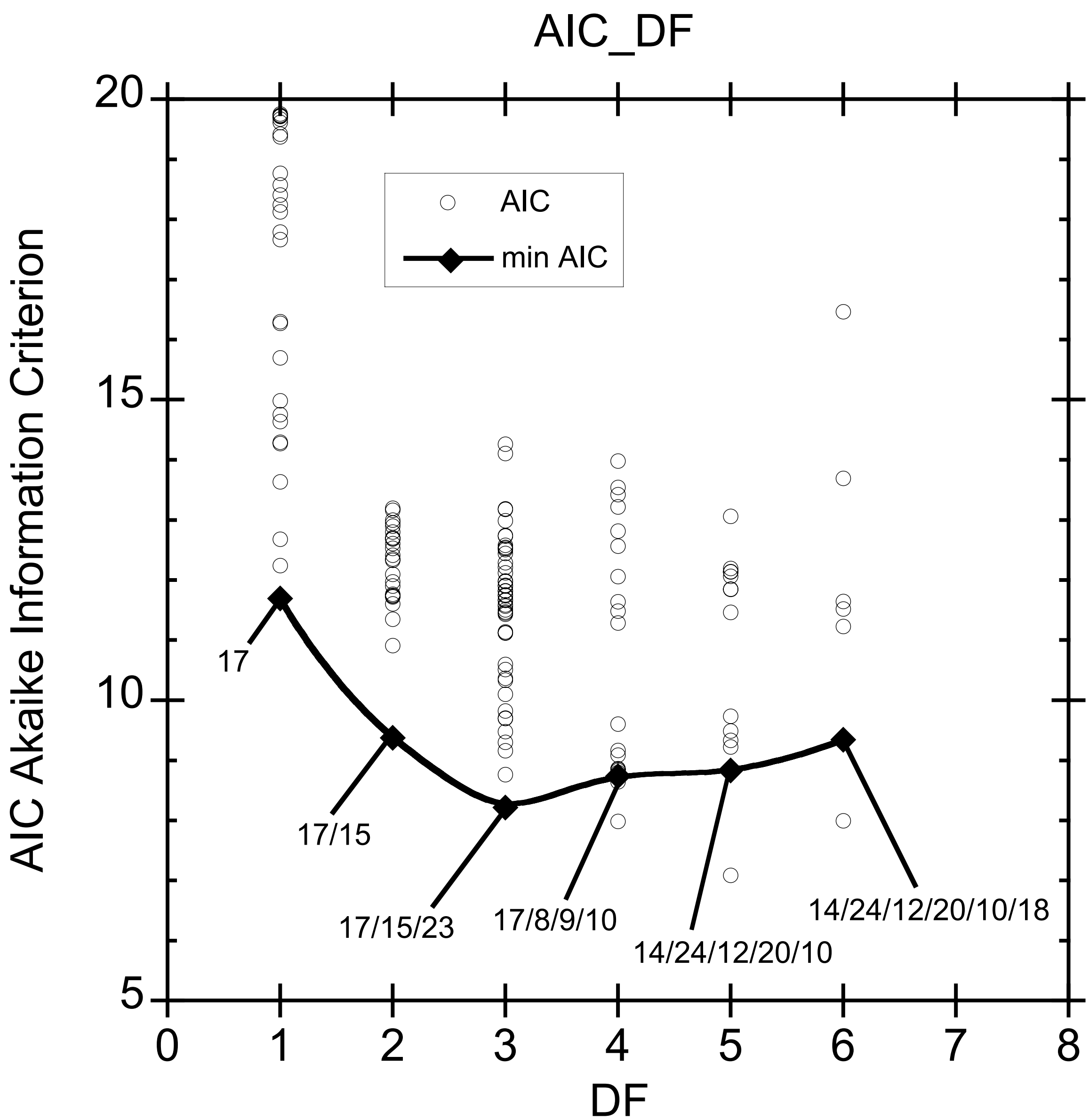


Figure E-7 Plot of AIC as a function of DF. The solid curve goes through the models that have the smallest AIC and reasonably small parameter uncertainties. The best model contains parameters 17, 15, and 23 corresponding to T_{rf}/W_{st}, H_{st}/D_{st}, and S_W, respectively. Models represented by open black circles that fall below the black curve have large parameter uncertainties.

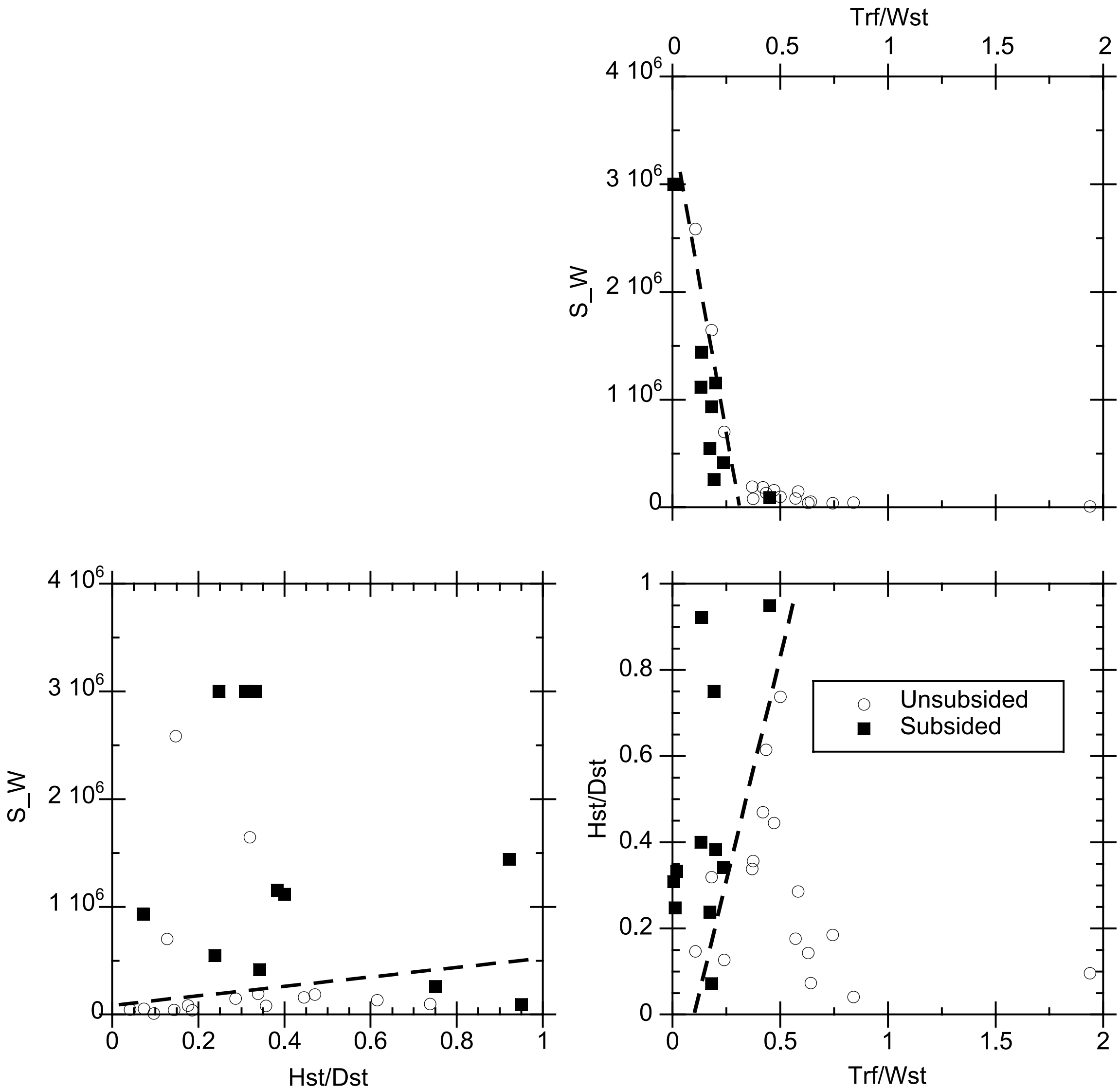


Figure E-8 Scatter plots of T_{rf}/W_{st} , H_{st}/D_{st} , and S_W . The dashed line are positioned to separate the subsided and unsubsided cases.

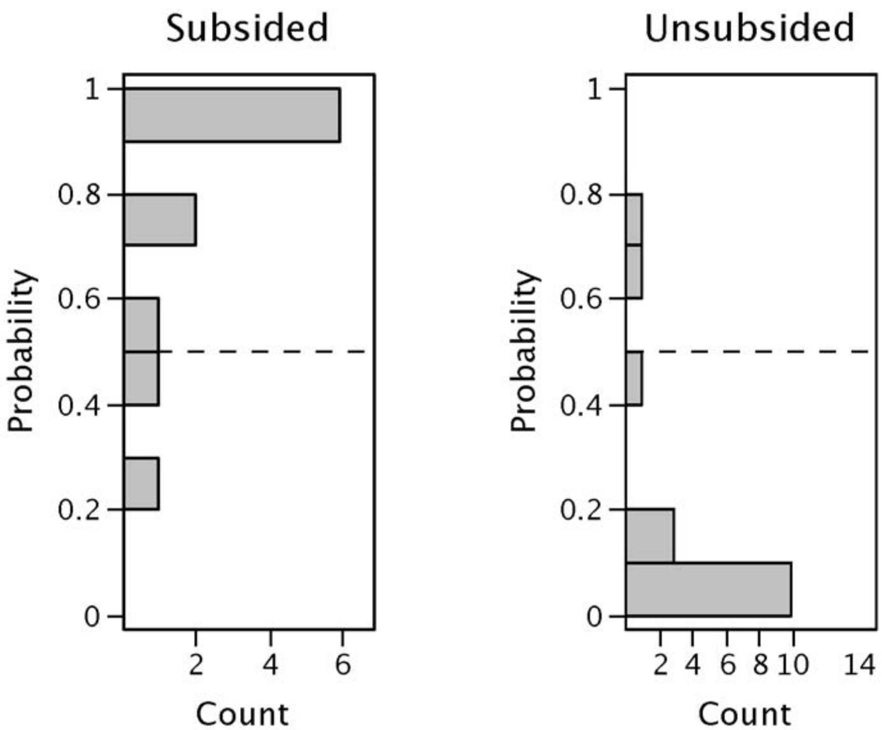


Figure E-9 Histograms of logistic regression estimated probability for subsided and unsubsided sites used in back analysis.